Math 241 Sample Problems for Exam 2

Question 1 Evaluate the following double integral:

$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy$$

Question 2 Find the volume of the solid in space which lies below the surface $z = 3 + \cos y$ and above the region in the xy–plane bounded by the curves $x = \pi$, $y = 0$, and $y = 2x$ by evaluating an appropriate double integral.

Question 3 Let R be the solid region bounded by the planes $x = 0$, $y = 0$, $z = 2$, and the paraboloid $z = x^2 + y^2$, in the first octant. Compute $\iiint_R x \, dV$.

Question 4 Find the volume determined by $z \leq 6 - x^2 - y^2$ and $z \geq \sqrt{x^2 + y^2}$.

Question 5 Convert the integral $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2 \, dz \, dx \, dy$ to spherical coordinates. Don't evaluate it.

Question 6 Evaluate $\iint_R \exp\left(\dfrac{y - x}{y + x}\right) dA$ where R is the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$, by using the change of variables $x = \frac{1}{2}(v - u)$, $y = \frac{1}{2}(u + v)$.

Question 7
a) Evaluate the line integral $\int_c x^2 y \, dx + xy^3 \, dy$, where $\mathbf{c}$ consists of the line segments from $(0,0)$ to $(3,3)$ and from $(3,3)$ to $(0,3)$.
b) Evaluate the line integral $\int_c (e^y + ye^x) \, dx + (e^x + xe^y) \, dy$, where $\mathbf{c}$ is the part of the graph $y = \ln 6x$ joining $(1/6, 0)$ to $(1/2, \ln 3)$.

Question 8 Find the area of the surface that is part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.